

# Consistency of cosmic strings with cosmic microwave background measurements

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In the context of SUSY GUTs, GUT scale cosmic strings formed at the end of hybrid inflation are compatible with currently available CMB measurements. The maximum allowed cosmic strings contribution to the CMB data constrains the free parameters (mass scales, couplings) of inflationary models. For F-term inflation either the superpotential coupling must be fine tuned or one has to invoke the curvaton mechanism. D-term inflation should be addressed in the framework of SUGRA. We find that the cosmic strings contribution is not constant, contrary to some previous results. Using CMB measurements we constrain the gauge and the superpotential couplings.

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Topological defects are the natural outcome of phase transitions accompanied by Spontaneously Symmetry Breaking (SSB) as the Universe expands. Among the various types of defects only Cosmic Strings (CS) are not dangerous to overclose the Universe. To get rid of the undesired defects, monopoles and domain walls, one usually employs the mechanism of inflation. Starting from a large gauge group in the context of Supersymmetric (SUSY) Grand Unified Theories (GUTs), in Ref. [1] there were considered all possible SSB schemes down to the Standard Model (SM) gauge group. Placing one or more inflationary eras to dilute the harmful stable defects, it was found that CS are always left behind. From the observational point of view however, strong constraints [2, 3] are placed on the allowed CS contribution to the angular power spectrum of the Cosmic Microwave Background (CMB) temperature anisotropies. This finding led some colleagues to conclude that CS, at least of the GUT scale, are ruled out, since their theoretical predictions seemed to be inconsistent with measurements. The aim of our study is to show that this is by no means the case. We find instead that the maximum allowed CS contribution to the CMB measurements places upper limits on the inflationary scale (which is also the CS energy scale), or equivalently on the coupling constant of the superpotential. Our analysis, in the framework of supersymmetric hybrid inflation, is given below first for F- and then for D-term type, where we also consider Supergravity (SUGRA) corrections, which turn out to be essential.

## A. F-term inflation

F-term inflation is based on the supersymmetric renormalisable superpotential

$$W_{\text{infl}}^{\text{F}} = \kappa S(\Phi_+ \Phi_- - M^2), \quad (1)$$

where  $S, \Phi_+, \Phi_-$  are three chiral superfields, and  $\kappa, M$  are two constants. We denote hereafter the superfield and its scalar component by  $S$ . Taking into account the one-loop radiative corrections to the scalar potential along the inflationary valley, the effective potential reads [4, 5, 6, 7]

$$V_{\text{eff}}^{\text{F}}(|S|) = \kappa^2 M^4 \left\{ 1 + \frac{\kappa^2 \mathcal{N}}{32\pi^2} \left[ 2 \ln \frac{|S|^2 \kappa^2}{\Lambda^2} + (z+1)^2 \ln(1+z^{-1}) + (z-1)^2 \ln(1-z^{-1}) \right] \right\}, \quad (2)$$

where  $z = |S|^2/M^2 \equiv x^2$ ,  $\Lambda$  is a renormalisation scale and  $\mathcal{N}$  stands for the dimensionality of the representation to which the complex scalar components  $\phi_+, \phi_-$  of the chiral superfields  $\Phi_+, \Phi_-$  belong.

On large angular scales, the main contribution to the CMB anisotropies is given by the Sachs-Wolfe effect. The quadrupole anisotropy has a contribution from the quantum fluctuations of the inflaton field and, in our model, a

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contribution from the CS network. The inflaton contribution (splitted into scalar and tensor parts) can be computed analytically, while the CS contribution is only derived from numerical simulations [8]. Their sum reads [7],

$$\left(\frac{\delta T}{T}\right)_{\text{Q-tot}}^2 \sim y_Q^{-4} \left(\frac{\kappa^2 \mathcal{N} N_Q}{32\pi^2}\right)^2 \left[ \frac{64N_Q}{45\mathcal{N}} x_Q^{-2} y_Q^{-2} f^{-2}(x_Q^2) + \left(\frac{0.77\kappa}{\pi}\right)^2 + 324 \right], \quad (3)$$

where  $x_Q^2 = |S_Q|^2/M^2$  (the index Q denotes the scale corresponding to the quadrupole anisotropy),  $y_Q^2$  is defined by

$$y_Q^2 = \int_1^{x_Q^2} \frac{dz}{zf(z)}, \quad (4)$$

and the number of e-foldings  $N_Q$  is

$$N_Q = \frac{4\pi^2}{\kappa^2 \mathcal{N}} \frac{M^2}{M_{\text{Pl}}^2} y_Q^2, \quad (5)$$

with  $M_{\text{Pl}}$  the reduced Planck mass [ $M_{\text{Pl}} = (8\pi G)^{-1/2}$ ], and  $f(z) = (z+1)\ln(1+z^{-1}) + (z-1)\ln(1-z^{-1})$ . The *l.h.s.* of Eq. (3) is normalised to the COBE data. For given  $\kappa$ ,  $N_Q$ , and  $\mathcal{N}$ , Eq. (3) can be solved numerically [7] to get  $x_Q$ , which is the only unknown quantity. Thus, we are able to compute  $y_Q$  and  $M$ . We find [7] that the mass scale  $M$  grows very slowly with  $\mathcal{N}$  and it is of the order of  $10^{15}$  GeV. Hereafter we use  $\mathcal{N} = 3$  since this is the most generic value in SUSY GUTs [7].  $M$  gives the scale of inflation as well as the mass scale of cosmic strings formed at the end of the inflationary era. The mass scale  $M$  is related to the coupling  $\kappa$  through

$$\frac{M}{M_{\text{Pl}}} = \frac{\sqrt{N_Q \mathcal{N}} \kappa}{2\pi y_Q}. \quad (6)$$

The CS contribution to the CMB, denoted by  $\mathcal{A}_{\text{CS}}$ , is a function of the coupling  $\kappa$ , or equivalently of the mass scale  $M$ . In Fig. 1 we show the inflaton and cosmic strings contributions with respect to  $\kappa$  and  $M$ .

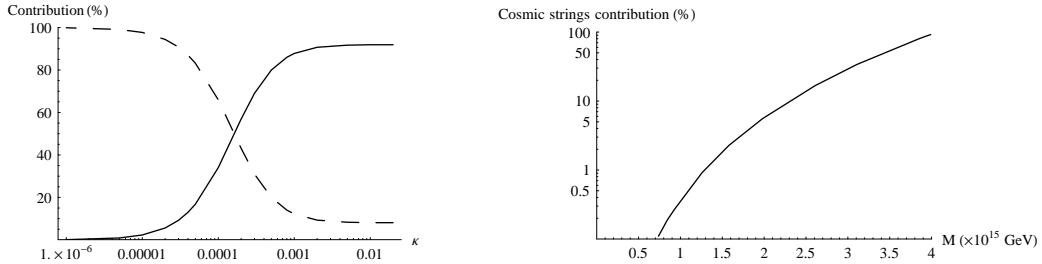


FIG. 1: On the left, cosmic strings (full line) and inflaton field (broken line) contributions to the CMB fluctuations as a function of the coupling  $\kappa$ . On the right, the cosmic strings contribution as a function of the mass scale of inflation  $M$ .

We find that  $\mathcal{A}_{\text{CS}}$  is consistent with the WMAP CMB data, namely [3]  $\mathcal{A}_{\text{CS}} \lesssim 9\%$ , provided

$$M \lesssim 2.2 \times 10^{15} \text{ GeV} \Leftrightarrow \kappa \lesssim 3 \times 10^{-5}. \quad (7)$$

Our finding is important: CS of the GUT scale are allowed. On the other hand, for this statement to be true, hybrid SUSY inflation loses some of its appeal since it is required some amount of fine tuning of its free parameter, namely  $\kappa$  should be of the order of  $10^{-5}$  or smaller. This constraint on  $\kappa$  is in agreement with the one given in Ref. [9]. The parameter  $\kappa$  is also subject to the gravitino constraint which imposes an upper limit to the reheating temperature, to avoid gravitino overproduction. Within SUSY GUTs, and assuming a see-saw mechanism to give rise to massive neutrinos, the inflaton field will decay during reheating into pairs of right-handed neutrinos. Using the constraints on the see-saw mechanism it is possible [6, 7] to convert the constraint on the reheating temperature to a constraint on the coupling parameter  $\kappa$ , namely  $\kappa \lesssim 8.2 \times 10^{-3}$ , which is clearly a weaker constraint.

The superpotential coupling  $\kappa$  is allowed to get higher values, namely it can approach the upper limit permitted by the gravitino constraint, if one employs the curvaton mechanism [10, 11]. Such a mechanism can be easily

accommodated within SUSY theories, where one expects to have a number of scalar fields. The extra curvaton contribution to the quadrupole anisotropy reads [7]

$$\left(\frac{\delta T}{T}\right)_{\text{curv}}^2 = y_Q^{-4} \left(\frac{\kappa^2 \mathcal{N} N_Q}{32\pi^2}\right)^2 \left[\left(\frac{16}{81\pi\sqrt{3}}\right) \kappa \left(\frac{M_{\text{Pl}}}{\psi_{\text{init}}}\right)\right]^2, \quad (8)$$

where  $\psi_{\text{init}}$  denotes the initial value of the curvaton field. For fixed  $\kappa$ , the CS contribution decreases rapidly as  $\psi_{\text{init}}$  decreases. Thus, the WMAP measurements lead to an upper limit on  $\psi_{\text{init}}$ , namely  $\psi_{\text{init}} \lesssim 5 \times 10^{13} (\kappa/10^{-2})$  GeV. This limit holds for  $\kappa$  in the range  $[5 \times 10^{-5}, 1]$ ; for lower values of  $\kappa$ , the cosmic strings contribution is always suppressed and thus lower than the WMAP limit.

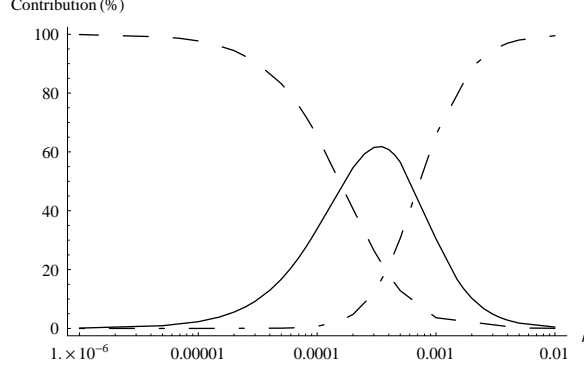


FIG. 2: Contribution from the three different sources to the CMB anisotropies as a function of the superpotential coupling  $\kappa$ , for  $\psi_{\text{init}} = 10^{13}$  GeV. The three curves show the contributions from cosmic strings (full line), inflaton (curve with broken line) and curvaton fields (curve with lines and dots).

Including SUGRA corrections to the F-term inflation we find [7] no difference in the calculated  $\delta T/T$ . This is expected since the value of the inflaton field is several orders of magnitude below the Planck scale.

## B. D-term inflation

D-term inflation is derived from the superpotential

$$W_{\text{infl}}^{\text{D}} = \lambda S \Phi_+ \Phi_- . \quad (9)$$

D-term inflation requires the existence of a nonzero Fayet-Illiopoulos term  $\xi$ , permitted only if an extra U(1) symmetry is introduced. As previously, we calculate in the SUSY framework the radiative corrections leading to the effective potential [7]

$$V_{\text{eff}}^{\text{D}}(|S|) = \frac{g^2 \xi^2}{2} \left\{ 1 + \frac{g^2}{16\pi^2} \left[ 2 \ln \frac{|S|^2 \lambda^2}{\Lambda^2} + (z+1)^2 \ln(1+z^{-1}) + (z-1)^2 \ln(1-z^{-1}) \right] \right\}, \quad (10)$$

where  $z = \lambda^2 |S|^2 / (g^2 \xi) \equiv x^2$ , with  $g$  the gauge coupling of the U(1) symmetry and  $\xi$  the Fayet-Illiopoulos term, chosen to be positive. We find that in the absence of the curvaton mechanism, the quadrupole anisotropy reads [7]

$$\left(\frac{\delta T}{T}\right)_{\text{Q-tot}} \sim \left\{ y_Q^{-4} \left(\frac{\lambda^2 N_Q}{16\pi^2}\right)^2 \left[ \frac{16 N_Q}{45} x_Q^{-2} y_Q^{-2} f^{-2}(x_Q^2) + \left(\frac{0.77g}{\sqrt{2\pi}}\right)^2 + 324 \right] \right\}^{1/2}, \quad (11)$$

where the number of e-foldings is

$$N_Q = \frac{2\pi^2}{\lambda^2} \frac{\xi}{M_{\text{Pl}}^2} \int_{z_{\text{end}}}^{z_Q} \frac{dz}{z f(z)}, \quad (12)$$

with  $z_{\text{end}} = \lambda^2 |S_{\text{end}}|^2 / (g^2 \xi)$ . Since inflation ends when either the symmetry is spontaneously broken or the slow roll conditions are violated, assuming the gravitino constraint on  $\lambda$ , we get [7]  $z_{\text{end}} \simeq 1$ . The *l.h.s.* of Eq. (11) is normalised to the COBE data, i.e.,  $(\delta T/T)_Q^{\text{COBE}} \sim 6.3 \times 10^{-6}$ .

We compute the mass scale of the SSB, given by  $M_D = \sqrt{\xi}$ , and find that the dependence of  $M_D$  on  $\lambda$  is very close to the one obtained for F-term inflation. The cosmic strings contribution is also very similar to the F-term case, implying that within the SUSY framework, the same conclusions hold, namely  $\lambda \lesssim 3 \times 10^{-5}$ .

However, the dependence of  $x_Q$  on the superpotential coupling  $\lambda$  results to a higher value of the inflaton field  $S_Q$  than in the F-term case, especially for large values of the gauge coupling  $g$ . This implies that the correct analysis has to be done in the framework of SUGRA. The SUSY analysis will be a limit of the SUGRA study for small values of the inflaton field. Some previous studies [12, 13] found in the literature kept only the first term of the radiative corrections. We find that it is necessary to perform the analysis using the *full* effective potential, which for minimal supergravity reads [7]

$$V_{\text{eff}}^{\text{D-SUGRA}} = \frac{g^2 \xi^2}{2} \left\{ 1 + \frac{g^2}{16\pi^2} \left[ 2 \ln \frac{|S|^2 \lambda^2}{\Lambda^2} \exp \left( \frac{|S|^2}{M_{\text{Pl}}^2} \right) + (z+1)^2 \ln(1+z^{-1}) + (z-1)^2 \ln(1-z^{-1}) \right] \right\}, \quad (13)$$

where  $z = [\lambda^2 |S|^2 / (g^2 \xi)] \exp(|S|^2 / M_{\text{Pl}}^2) \equiv x^2$ . The constant term in the potential is identical to the SUSY case, but its first derivative is modified [7] by the factor  $(1 + |S|^2 / M_{\text{Pl}}^2)$ . The number of e-foldings is [7]

$$N_Q = \frac{2\pi^2}{g^2} \int_1^{x_Q^2} \frac{W(cz)}{z^2 f(z) [1 + W(cz)]^2} dz, \quad (14)$$

where  $W(x)$  denotes the “W-Lambert function” defined by  $W(x) \exp[W(x)] = x$ , and  $c \equiv (g^2 \xi) / (\lambda^2 M_{\text{Pl}}^2)$ . The number of e-foldings  $N_Q$  is thus a function of  $c$  and  $x_Q$ , for  $g$  fixed. Setting  $N_Q = 60$  we obtain a numerical relation between  $c$  and  $x_Q$  which allows us to construct a function  $x_Q(\xi)$  and express the three contributions to the CMB only as a function of  $\xi$ . Thus, we solved

$$\left( \frac{\delta T}{T} \right)_{Q-\text{tot}} \sim \frac{\xi}{M_{\text{Pl}}^2} \left\{ \frac{\pi^2}{90g^2} x_Q^{-4} f^{-2}(x_Q^2) \frac{W(cx_Q^2)}{[1 + W(cx_Q^2)]^2} + \left( \frac{0.77g}{8\sqrt{2}\pi} \right)^2 + \left( \frac{9\pi}{4} \right)^2 \right\}^{1/2}, \quad (15)$$

to obtain  $\xi$ , and then the CS contributions for a given value of  $g$ . Our results are summarised in Fig. 3

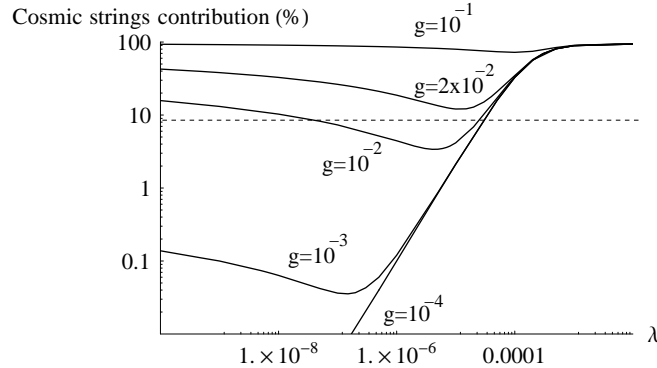


FIG. 3: Cosmic strings contribution to the CMB fluctuations as a function of the superpotential coupling  $\lambda$  for different values of the gauge coupling  $g$ . The maximal contribution allowed by WMAP is represented by a dotted line.

Within this approach for D-term inflation, our findings are different than within the framework of SUSY [7], unless  $\lambda \gtrsim 10^{-3}$  or  $g \lesssim 10^{-4}$ ; in these cases we are in the SUSY limit. The CS contribution to the CMB turns out to be model-dependent, with however the robust results that CS contribution is not constant, nor is it always dominant, in contradiction to Ref. [12]. This implies that contrary to what is often assumed, D-term inflation is still an open possibility. Our analysis [7] shows that  $g \gtrsim 1$  necessitates multiple-stage inflation, since otherwise we cannot have  $N_Q = 60$ , while  $g \gtrsim 2 \times 10^{-2}$  is incompatible with the allowed CS contribution to the WMAP measurements. For  $g \lesssim 2 \times 10^{-2}$ , we can also constrain the superpotential coupling  $\lambda$  and get  $\lambda \lesssim 3 \times 10^{-5}$ . This limit was already found in the SUSY framework [7] and it is in agreement with the finding  $\lambda \lesssim \mathcal{O}(10^{-4} - 10^{-5})$  of Ref. [13]. However, we disagree with the result of Ref. [13] stating that  $\lambda$  can be of order 1 using the curvaton mechanism, since this is forbidden by the gravitino constraint [7]. This upper bound on  $\lambda$  is also in agreement with the one found in Ref. [9] in the context of P-term inflation. SUGRA corrections impose [7] in addition a lower limit to the coupling  $\lambda$ . If for

example  $g = 10^{-2}$ , the cosmic strings contribution imposes  $10^{-8} \lesssim \lambda \lesssim 3 \times 10^{-5}$ . This constraint induced by the CMB measurements is expressed as a single constraint on the Fayet-Iliopoulos term  $\xi$ , namely  $\sqrt{\xi} \lesssim 2.3 \times 10^{15}$  GeV.

We would like to bring to the attention of the reader that in the above study we have neglected the quantum gravitational effects, which would lead to a contribution to the effective potential, even though  $S_Q \sim \mathcal{O}(10M_{\text{Pl}})$ . Our analysis is however still valid, since the effective potential given in Eq. (13) satisfies the conditions [14]  $V(|S|) \ll M_{\text{Pl}}^4$  and  $m_S^2 = d^2V/dS^2 \ll M_{\text{Pl}}^2$ , and thus the quantum gravitational corrections  $[\Delta V(|S|)]_{\text{QG}}$  are negligible when compared to the effective potential  $V_{\text{eff}}^{\text{D-SUGRA}}$ .

In conclusion, our study has shown that cosmic strings are allowed within SUSY GUTs and they may have an important cosmological rôle. We proved that CS formed at the GUT scale are consistent with the recent CMB measurements. The implications of our findings are important. High energy physics tells us that cosmic strings are expected to be formed at the end of hybrid inflation [1]. These objects may lead to important cosmological consequences. Measurements from the realm of cosmology on the other hand constrain the possible rôle of cosmic strings, leading to constraints on the free parameters of the employed SUSY/SUGRA models. Thus, cosmology gives us, in return, information about high energy physics. This is indeed the beauty of this subject.

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